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LETTER TO THE EDITOR

Renormalisation of the long-range 'true' self-avoiding walk

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Abstract. We show that the 'true' self-avoiding walk with long-range interactions leads to a renormalisable field theory. We compute the characteristic exponent of the gyration radius up to first order in the difference between the upper critical dimension and space dimensionality, and compare it with the estimates based on Flory-like arguments.

The 'true' self-avoiding walk (TSAW) introduced by Amit *et al* (1983) is a convenient toy model of aggregation processes. While retaining the characteristic memory effects of aggregation, it is very simple to simulate and has a non-trivial behaviour at low dimensionality. On the other hand, it is easy to analyse it by field theoretical perturbation expansions or by the analogue of high-temperature series. Therefore, although it does not probably describe any real physical process (but cf Bulgadaev and Obukhov 1983), it has been treated by several methods as an arena where new techniques and concepts could be easily tested (for a review, see Peliti and Pietronero 1985).

One of the authors (Zhang 1985) has recently introduced a generalisation of the TSAW where the repulsion acting on the walker from all sites it has previously visited has long range, decaying with the distance r as $|r|^{-(d-2\alpha)}$. A Flory argument (in the spirit of Pietronero (1983)) yielded as a result that the upper critical dimensionality d_c (above which the walker undergoes Brownian motion) is given for such a model by

$$d_c = 2 + 2\alpha \tag{1}$$

and that the exponent ν which describes the dependence $R \propto N^{\nu}$ of the radius of gyration as a function of the number of steps is given by

$$\nu_F = 2/(d + 2 - 2\alpha). \tag{2}$$

This gives in particular $d_c = 4$ and $\nu = 2/d$ for Coulomb interactions ($\alpha = 1$). Flory arguments are known to give rather good estimates of exponents for self-repelling chains with short-range interactions (cf de Gennes 1979). The situation is not so clear in the presence of long-range interactions, such as in polyelectrolytes. Similarly, whereas Pietronero's argument probably yields the correct exponent for the one-dimensional TSAW, it is not obvious whether it should work with the same success with long-range interactions. Monte Carlo simulations (Zhang 1985) show nevertheless the Flory–Pietronero-like estimates to be exact within 10%.

We apply here field theoretical renormalisation group methods to the same problem and find the model to correspond to a renormalisable field theory. We obtain confirmation of the value of d_c (equation (1)). The exponents are not regular as a function of

α around $\alpha = 0$. To first order in $\varepsilon = d_c - d$, the exponent ν has the expansion:

$$\nu = \tfrac{1}{2} + \tfrac{1}{3}\varepsilon, \tag{3}$$

which does not agree with the first-order expansion of (2):

$$\nu_F = \tfrac{1}{2} + \tfrac{1}{8}\varepsilon. \tag{4}$$

We have in fact considered a slightly more general model, which also encompasses the random walk in a random environment, for a case in which the correlations of the quenched randomness decay as $|r|^{-(d-2\alpha)}$. The upper critical dimension is correctly given by (1). The first order in the ε expansion of ν is given by

$$\nu = \tfrac{1}{2} + \frac{\alpha}{2(1+\alpha)}\,\varepsilon. \tag{5}$$

The model is defined as follows. We consider a walker whose position $\boldsymbol{R}(t)$ evolves according to the equation:

$$\frac{\mathrm{d}\boldsymbol{R}(t)}{\mathrm{d}t} = -g_1 \nabla\phi(\boldsymbol{R}(t), t) + \boldsymbol{\eta}(t), \tag{6}$$

where $\boldsymbol{\eta}(t)$ is a Gaussian white noise process of average zero and of correlation function

$$\langle \eta_\alpha(t)\eta_\beta(t')\rangle = 2D\delta_{\alpha\beta}\delta(t-t'). \tag{7}$$

The potential $\phi(\boldsymbol{r}, t)$ is expressed as a function of the density $\rho(\boldsymbol{r}, t)$ of points which have been visited by the walker up to time t by

$$\phi(\boldsymbol{r}, t) = \int \mathrm{d}^d\boldsymbol{r}'\, K(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}', t) \tag{8}$$

and the kernel $K(\boldsymbol{r})$ is given by

$$K(\boldsymbol{r}) = \int \frac{\mathrm{d}^d\boldsymbol{k}}{(2\pi)^d}\,\frac{\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})}{|\boldsymbol{k}|^{2\alpha}} \simeq |\boldsymbol{r}|^{-(d-2\alpha)}. \tag{9}$$

The integral is regularised by a suitable ultraviolet cutoff. Using standard arguments (Grasberger and Scheunert 1980) one formulates the problem as a field theory defined by the action

$$H = H_0 + H_1, \tag{10}$$

where, up to irrelevant terms, one has:

$$H_0 = \int \mathrm{d}t \int \mathrm{d}^d\boldsymbol{r}\,\bar{\psi}(\boldsymbol{r}, t)[-\partial\psi/\partial t + D\nabla^2\psi(\boldsymbol{r}, t)], \tag{11}$$

$$H_1 = -g_1 \int \mathrm{d}t \int \mathrm{d}^d\boldsymbol{r}\,\bar{\psi}(\boldsymbol{r}, t)\nabla\psi(\boldsymbol{r}, t)\cdot\nabla\left[\int \mathrm{d}^d\boldsymbol{r}'\, K(\boldsymbol{r}-\boldsymbol{r}') \times \int_0^t \mathrm{d}t'\,\bar{\psi}(\boldsymbol{r}', t')\psi(\boldsymbol{r}', t')\right]. \tag{12}$$

One may easily read off in equations (11) and (12) the dimension of the coupling constant and therefore the upper critical dimension. Feynman rules for the perturbative expansion of the Fourier (space) and Laplace (time) transform of the end point probability distribution are easily obtained. The walk is represented by a full, directed line, in which a wavenumber $\boldsymbol{p}$ flows. The interactions are represented by broken lines, joining points of the solid line and carrying wavevector $\boldsymbol{q}$ in the direction *opposite* to the walk. Wavevector conservation is assumed at each vertex. Each segment of the full line represents a factor $(\mu + Dp^2)^{-1}$, where μ is the Laplace transform parameter and $\boldsymbol{p}$ is the wavenumber which flows in that segment. We define a general interaction vertex as in figure 1. The point A is assumed to come *earlier* than point B if one moves along the walk. The general interaction contribution will be of the form:

$$\gamma(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{p}_1) = [g_1(\boldsymbol{p}_1 \cdot \boldsymbol{q}) + g_2(\boldsymbol{p}_1 \cdot (\boldsymbol{p}_1 + \boldsymbol{q})) + g_3(\boldsymbol{p}_1 \cdot \boldsymbol{p})]|\boldsymbol{q}|^{-2\alpha}. \tag{13}$$

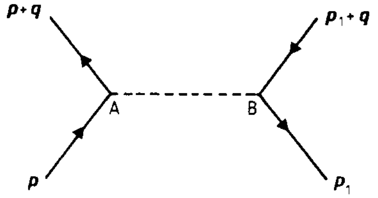

Figure 1. The basic interaction vertex. Point A is assumed to come earlier than point B along the directed line.

For our model, only g_1 is different frm zero. If we were instead modelling a random walk in a random environment (RRW) where correlations of the quenched velocity field are proportional to $K(\boldsymbol{r})$, we would obtain

$$g_1 = g_3 = -g \qquad (g > 0) \qquad g_2 = 0. \tag{14}$$

Renormalisation of this field theory is fairly straightforward. One notices that the only primitively divergent graphs must contain no more interaction (broken) lines than loops. This ensures that the renormalised vertex still contains a $|\boldsymbol{q}|^{-2\alpha}$ factor, where $\boldsymbol{q}$ is the transferred wavevector. Some diagrams that contribute to the vertex renormalisation in the short-range TSAW do not contribute here. Therefore the exponents turn out to be discontinuous as $\alpha \to 0$. One introduces a wavefunction renormalisation constant Z and dimensionless renormalised coupling constants u_i $(i = 1, 2, 3)$ in a way analogous to Obukhov and Peliti (1983). One then computes the corresponding exponent function η and Wilson functions W_i $(i = 1, 2, 3)$ obtaining as a result:

$$\eta = \left.\frac{\partial \ln Z}{\partial \ln \kappa}\right|_0 = \frac{u_1}{1+\alpha} - \frac{\alpha u_2}{1+\alpha} - u_3 \tag{15}$$

$$W_1 = \left.\frac{\partial u_1}{\partial \ln \kappa}\right|_0 = -\varepsilon u_1 + \frac{3u_1^2}{2(1+\alpha)} - \frac{1+2\alpha}{2(1+\alpha)} u_2^2 - \frac{2\alpha}{1+\alpha} u_1 u_2 - \frac{5+4\alpha}{2(1+\alpha)} u_1 u_3 - \frac{u_2 u_3}{2(1+\alpha)} \tag{16}$$

$$W_2 = \left.\frac{\partial u_2}{\partial \ln \kappa}\right|_0 = -\varepsilon u_2 - \frac{2+3\alpha}{2(1+\alpha)} u_2^2 - \frac{u_1 u_2}{1+\alpha} - \frac{1+3\alpha}{1+\alpha} u_2 u_3 \tag{17}$$

$$W_3 = \left.\frac{\partial u_3}{\partial \ln \kappa}\right|_0 = -\varepsilon u_3 - 2u_3^2 + \frac{u_1 u_3}{1+\alpha} - \frac{1+2\alpha}{1+\alpha} u_2 u_3. \tag{18}$$

Here κ is the renormalisation wavenumber. One notices that, contrary to the case of the short-range TSAW, no u_2, u_3 couplings are generated by the renormalisation if they are zero at the beginning. The relevant fixed points to first order in ε are given by

$$\text{TSAW: } u_1^* = \tfrac{2}{3}(1+\alpha)\varepsilon \qquad u_2^* = u_3^* = 0 \tag{19}$$

$$\text{RRW: } u_1^* = u_3^* = -\frac{1+\alpha}{1+2\alpha}\varepsilon \qquad u_2^* = 0. \tag{20}$$

The results (equations (3) and (5)) follow if one considers that the exponent ν is connected to η via a scaling law of the form $\gamma/\nu = 2-\eta$, where the susceptibility exponent γ is identically equal to 1 in view of probability conservation.

We have shown in conclusion the tractability of the problem of the long-range TSAW by the traditional methods of renormalised field theory. While the exponents so obtained disagree with estimates based on Flory-like approaches, they are consistently larger, as in previous instances. Nevertheless it would be interesting to perform Monte Carlo simulations for such a model with a suitably chosen α such that, say, two dimensions were just below the upper critical dimension, to check whether the striking success of Flory arguments persists even in a domain where one would reasonably rely on ε expansions.

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